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АРХІТЕКТУРА ТА БУДІВНИЦТВО

NUMERICAL AND ANALYTICAL SOLUTION OF THE PROBLEM OF HEAT TRANSFER UNDER BOUNDARY CONDITIONS OF THE THIRD KIND

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In recent years, considerable attention has been paid to solving related problems, which most fully take into account the actual conditions of heat transfer in tubular heat exchangers. The papers [1-3] disclosed related problems of heat transfer in laminar fluid flow in pipes and channels, which can be used to assess the role of this factor. The role of non-Newtonian properties in solving adjoint problems is studied in the monograph [4]. The data available in these works make it possible to quantify the errors due to the fact that the conjugacy of the problem and the setting of the third kind of boundary conditions on the wall are not taken into account. The latter means that the heat flux density is assumed to be proportional to the temperature difference between the outer surface of the wall T_{w1} and the environment T_{en}:

$$q_w = \alpha_1 \cdot (T_{w1} - T_{en}); \qquad (1)$$

where α_1 - coefficient of heat transfer from the outer surface of the pipe wall to the environment.

The values α_1 and T_{en} are considered given; in the general case, they are functions of the coordinates on the pipe surface. The statement of such functions does not change the fundamental aspect of the problem, but only complicates the calculation. Due to small temperature gradients along the flow for highly viscous liquids, it is advisable to introduce one more assumption: the temperature field in the pipe wall is one-dimensional; that is, the heat transfer occurs only along the normal to its surface, and the heat transfer along the wall is small. Setting the boundary conditions of the third kind on the inner wall of the pipe is equivalent to replacing the solution of the adjoint problem with the solution of the problem of heat transfer only in the liquid.

Possible errors in such substitution should be evaluated. So, in the work of A.V. Lykov [1,] to estimate the conjugation conditions, the Brun number Br' is introduced, and for Br' ≤ 0.1, the problem is recommended to be solved without conjugation. For a flow in a pipe, the Brun number can be approximately estimated as:

$$Br' = \frac{\lambda_l}{\lambda_w} \cdot \frac{\delta_w}{d} \cdot Pe^{0,33} \tag{2}$$

where δ_w – pipe wall thickness, d –pipe diameter, λ_l, λ_w - coefficient of thermal conductivity, respectively, of the liquid, wall; Pe - Peclet criterion.

With the usual values for pipes *$\lambda l/\lambda w = \bar{\lambda}$* and *$\delta w/d$* number *$Br' \leq 10\text{-}2$.*

In [2], the conjugate problem for a power-law fluid (the Ostwald-de Wiel rheological model) was solved under the condition that the temperature on the outer wall of the pipe changes according to an arbitrary law. The parameter was introduced as an adjoint criterion:

$$\mathrm{K_s} = \bar{\lambda}/\ln \mathrm{R_1} \tag{3}$$

where R_1 - the ratio of the outer and inner radii of the pipe.

For $K_s \rightarrow \infty$, the problem is reduced to boundary conditions of the first kind at a variable wall temperature along the length. The data calculated for this case are consistent with those known in the literature. An estimate of the Ks value for heat transfer conditions in laminar flow in pipes indicates that $K_s > 0.01$. In [4], the factors influencing the degree of conjugation of problems are summarized in a table, which shows that for non-Newtonian fluids exhibiting pseudoplastic properties, the role of conjugation conditions is less than for Newtonian ones. The quantitative assessment of the conjugation conditions is considered in detail for the conditions of heat transfer between two fluids separated by a thin plate. It allows one to calculate the relative error in the value of the local heat flux density, which is due to the fact that the conjugacy of the problem is not taken into account. If we use this estimate, we can conclude that for liquids with $n=1/m<1$ (where m is the non-linearity index of the Bulkley-Herschel model) in the most unfavorable situations in terms of error, the latter will not exceed 5%, and for non-Newtonian fluids, it will be even smaller. In [3], adjoint problems for the flow of nonlinear viscoplastic fluids in plane-parallel channels were studied. From the above estimates, if extended to round pipes, it follows that the resulting errors, when conjugations are not taken into account, are within the accuracy of numerical calculations.

There are few works under boundary conditions of the third kind. In [4], the problem of heat transfer was solved by an approximate integral method for these conditions, taking into account the dissipation of the energy of motion. The significant influence of the number Bi, introduced as a parameter that characterizes the conditions at the boundary, on the local value of Nu is indicated. More fully, including the initial hydrodynamic sector, the problem for Newtonian fluids is considered in [5]. The solution of the hydrodynamic problem was obtained by the Galerkin-Kantorovich method, and the thermal problem was obtained by the numerical method. It follows from the data presented that the role of the hydrodynamic sector somewhat increases with a decrease in the Bi number (Biot criterion). Thus, the ratio of the Nusselt number with a developing velocity profile to the analytical value with a developed velocity profile for Bi = 100 (which practically coincides with $Bi \rightarrow \infty$) is 1.15 at $\frac{1}{Pe} \cdot \frac{Z}{d} = 10^{-3}$, and for Bi = 2 o it is equal to 1.25. Data refer to the Prandtl number Pr=10. (Note that the Oz axis coincides with the axis of the pipe). With an increase in $\frac{1}{Pe} \cdot \frac{Z}{d}$, as well as Prandtl numbers (Pr), the role of the initial hydrodynamic section decreases and at Pr ≥100 and $\frac{1}{Pe} \cdot \frac{Z}{d} > 10^{-3}$ within the accuracy of numerical calculations becomes insignificant, which is similar to the first boundary value problem.

The effect of mechanical energy dissipation on heat transfer for a Newtonian fluid was studied in [6]. The problem was solved by the usual method of separation of variables, three eigenvalues were tabulated, as well as the expansion coefficients for Bi, equal to 40.4 and 1, respectively. From the data on the limiting Nusselt numbers, it follows that the latter change from zero at Bi = 0 to 9, 6 for Bi→∞; in this case, the local number Nu calculated from the heat transfer coefficient on the wall is 9.6, regardless of the number Bi. Based on the analysis of the data obtained, a physically obvious circumstance is indicated that, as Bi decreases, the role of energy dissipation increases. The results of solving the problem of heat transfer for non-Newtonian fluids, taking into account the dissipation of the energy of motion, as is known, are not available [6].

The purpose of this work is to analytically determine the (local) total Nusselt number as a function of the usual local number Nu and the Biot criterion (Bi), as well as to substantiate the numerical method of solution and the results obtained for the heat transfer problem under boundary conditions of the third kind. At the same time, a numerical estimate of the value of Bi is established, according to which the problem with boundary conditions of the third kind can be reduced to a problem with boundary conditions of the first kind.

It is known that the Biot number:

$$Bi = K' \cdot \frac{d}{\lambda}, \tag{4}$$

Where $(K')^{-1} = \frac{d}{2\lambda_w} \cdot ln\left(\frac{d_1}{d}\right) + \frac{d}{\alpha_1 \cdot d_1}$; *K' - local heat transfer coefficient from the inner surface of the wall to the environment; λ, λ_w - coefficient of thermal conductivity of the environment and wall material, respectively; α1 - heat transfer coefficient of the outer surface of the pipe; d, d_1 - inner and outer diameters of the pipe.*

The number Bi and the ambient temperature, which are directly included in the dimensionless variables of the heat transfer problem, are considered given.

The total Nusselt number (local value) $Nu_k=K{\cdot}d/\lambda$ is related to the usual local Nusselt number $Nu_\alpha=\alpha{\cdot}d/\lambda$ by the known relation:

$$\frac{1}{Nu_k} = \frac{1}{Nu_\alpha} + \frac{1}{Bi} \tag{5}$$

In this case we have:

$$Nu_k = Nu_\alpha \cdot Bi/(Nu_\alpha + Bi), \tag{6}$$

that is, always $Nu_k < Nu_\alpha$, and $\frac{1}{K} = \frac{1}{\alpha} + \frac{1}{K\prime}$, where K is the local heat transfer coefficient from the fluid flowing through the pipe to the environment. Formula (6) is also valid with respect to the mean values of the numbers ($\overline{Nu}$. If Bi→∞, then $Nu_k = Nu_\alpha$ the problem of heat transfer under boundary conditions of the third kind is reduced to the problem under boundary conditions of the first kind; if Bi = 0, which corresponds to a thermally insulated wall, then Nu_k =0, and Nu_α is a finite value.

Various methods are known for solving the problem of heat transfer under boundary conditions of the third kind. Due to the cumbersomeness of the solution by the method of separation of variables (the eigenvalues and eigenfunctions in the Sturm-Lauville problem in this case depend on the number Bi), numerical methods have become widespread [5]. For non-Newtonian fluids, this circumstance is of particular importance, since the eigenvalues and all parameters of the Sturm-Lauville problem are, in addition, a function of the radius of the jet stream and the non-linearity index (n=1/m) present in the (rheological) Bulkley-Herschel model. For this reason, the solution of the heat transfer problem under boundary conditions of the third kind, as a rule, is not constructed, and the subsequent analysis is based on the results of the numerical solution provided in [7, 8]. The numerical solution method is also considered in detail in [6].

In Fig.1, the accuracy of the numerical method is estimated in comparison with the analytical one [8], which was used in solving heat transfer problems for boundary conditions of the first and third kind. The results of the temperature field calculations on a PC coincide within the limits of deviations, not exceeding (2…3)%, which indicates the high accuracy of the numerical method.

On fig. 1 introduced notations: T – liquid temperature, T_w – wall temperature, r_w – wall radius, r – radius (current), counted from the Oz axis of the cylindrical pipe, m – non-linearity index of the Bulkley-Herschel model, $\alpha = r_\alpha / r_w$ – dimensionless radius cores (r_α) of the jet stream; q_w is the heat flux through the pipe wall.

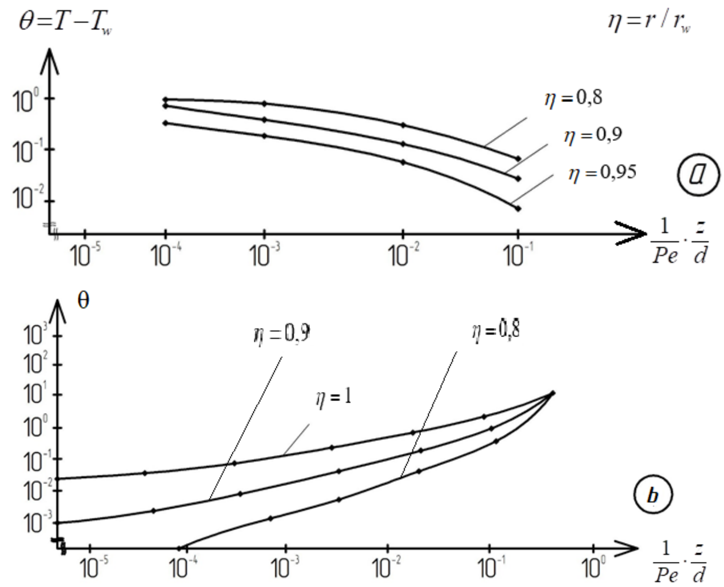

Fig. 1. **Results of calculation of the temperature field at m = 2, α = 0.5:**
a - T_w=const; b - q_w=const; curves are the results of the analytical solution [6], dots are the results of the numerical calculation on the PC of this work.

Next, we estimate the value of Bi for which the problem with boundary conditions of the third kind can be reduced to a problem with boundary conditions of the first kind.

It was shown in [5, 9] that for Newtonian fluids at Bi ≥40, the difference in heat transfer parameters is insignificant. An analysis of the obtained numerical results on a PC shows that for non-Newtonian liquids, even at Bi = 40, the difference in the average liquid temperature $\bar{\theta}$ and Nu numbers is within the accuracy of numerical calculations, although certain differences in the values of $\overline{\theta_w}$ (Fig. 2) still take place at small lengths; they disappear at Bi ≈ 60…70. The data in fig. 2 refer to m = 2 and α = 0.5, for other values of m and α the changes are insignificant.

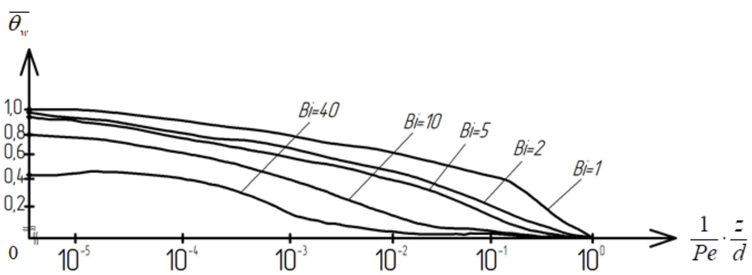

Fig. 2. **Changes in the dimensionless temperature of the inner wall of the pipe ($\overline{\theta_w}$) along its length at m = 2, α = 0.5, $Br_0^* = 0$ (Br_0^* is the Brun number [1]), $\theta = (T - T_w)/T_w/$.**

The local values of Nu_k (total Nusselt number) depending on the Biot criterion (Bi) are analytically obtained for solving the problem of heat transfer in a circular tube under boundary conditions of the third kind. The temperature field of a circular tube is calculated by numerical methods and the accuracy of such calculations is estimated in comparison with analytical approaches to solving heat transfer problems under limiting conditions of the first and third kind. A numerical estimate is made of the value of Bi, at which the problem with boundary conditions of the third kind can be reduced to a problem with boundary conditions of the first kind. The results of the study can be used to improve existing engineering methods for calculating heat transfer problems in heat exchange devices (apparatuses).

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